Cambridge Assessment International Education

Cambridge International AS & A Level

THINKING SKILLS **9694/33**

Paper 3 Problem Analysis and Solution **October/November 2023**

2 hours

You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
Calculator

INSTRUCTIONS

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
  Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
  In most questions, full marks will be awarded for a correct answer without any working. In some questions, however, you will not be awarded full marks if working needed to support an answer is not shown.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages.

1 Visits to sites in Antarctica by tourists are strictly controlled. The rules state that not more than one ship may visit each site each day, and not more than 100 tourists may land at each site each day. Ships vary in size – sometimes just one person might land.

Ships only visit sites where everyone on board may land, although sometimes a few people do not do so. There is a short summer season each year, and the unpredictable weather means that landings sometimes need to be rearranged or cancelled. Ships never go to the same site twice on a single cruise.

Each site has a fixed maximum number of tourists per season, and a record is kept of the actual number of visits to the sites. All have spectacular scenery, but some have points of special interest such as historic huts (H) and penguin colonies (P).

It is towards the end of the season and there are only two ships still there on otherwise typical cruises: Borchgrevink is carrying 97 tourists and Shirase has 53. Each ship has done two landings. The two southernmost sites, W and Y, have just been closed by frozen sea.

Site	*Special interest*	*Maximum tourists per season*	*Borchgrevink tourists to date*	*Shirase tourists to date*	*Total to date*
A		1000	94	52	972
C		2000	93		1696
D	P	500			444
F	H	1500			914
G	H P	1000			953
K	P	1500			1402
T	H	2000			1863
W		500		51	54
Y		500			46

(a) What is the minimum number of days that site C must have been open this season? [2]

(b) Which sites might it be possible for Borchgrevink to visit tomorrow? [1]

(c) What is the largest group that could have visited Y, if W is the site visited by the smallest number of ships? [2]

Both ships will visit just two more sites on their current cruises. They want their tourists to have the chance to see both a historic hut and a penguin colony.

(d) Give an example of which sites each ship should visit. [2]

(e) Estimate how many tourists there have been in total this season. State any assumptions you have made. [3]

[Turn over for Question 2]

2 A new form of long jump competition is being trialled. In the first round of the event, each athlete has up to three attempts to jump a certain distance. The rules are:

- Once an athlete has succeeded in jumping the distance, the athlete does not have any more attempts in the round.
- Any athlete who fails to jump the required distance in three attempts takes no further part in the event.

In the second round, the successful athletes from the first round each have up to three attempts to jump a new, longer distance. The same rules still apply. This process continues until only one athlete remains in the event.

If two (or more) athletes fail in three attempts at the same distance, their final positions in the event are determined by their total numbers of fails in the whole event: the athlete with fewer fails is placed higher. (Assume that there is never a tie.)

The distance set for the first round is decided by the organisers of the event, but in subsequent rounds the distance always increases by 0.2 m.

The results for a recent event with five athletes are given in the following table.

Athlete	*Distance (in metres)*				
	5.0	*5.2*	*5.4*	*5.6*	*5.8*
Matt	✘✓	✘✓	✘✘✓	✘✓	✘✘✘
Nathan	✓	✘✓	✘✓	✘✘✓	✘✘✘
Ollie	✘✘✓	✘✘✘	–	–	–
Pete	✘✓	✘✓	✘✘✘	–	–
Quentin	✘✘✘	–	–	–	–

This shows that, for example, Ollie had 2 fails and then 1 success at 5.0 m and 3 fails at 5.2 m, so he had no further attempts and a total of 5 fails.

(a) Who won this event? Justify your answer. [1]

In a second event involving these five athletes, Matt came 1st and Nathan came 2nd, both with a longest jump of 5.6 m. Ollie, Pete and Quentin came 3rd, 4th and 5th respectively, each with a longest jump of 5.4 m. The starting distance was 5.0 m. Nathan had a total of 5 fails and Pete had a total of 8 fails.

(b) Draw up a possible table, similar to the one above, to show this information. [3]

On another occasion, eleven athletes took part in a long jump event and the results of ten of these athletes are shown in the table below. By mistake, athlete Ken was omitted from this table. In fact, he came 6th.

Athlete	*Distance (in metres)*								
	5.0	*5.2*	*5.4*	*5.6*	*5.8*	*6.0*	*6.2*	*6.4*	*6.6*
Adi	✓	✓	✘✓	✘✓	✘✘✓	✘✘✓	✘✘✘	–	–
Ben	✓	✘✓	✓	✓	✘✓	✓	✘✓	✘✘✓	✘✘✘
Cal	✘✓	✓	✘✘✓	✘✘✓	✘✘✘	–	–	–	–
Den	✓	✘✓	✓	✘✓	✘✘✓	✘✘✘	–	–	–
Eric	✘✘✓	✘✓	✘✓	✘✘✓	✘✓	✘✓	✘✘✓	✘✓	✘✘✘
Fran	✘✓	✘✘✓	✘✘✓	✘✘✘	–	–	–	–	–
Greg	✘✘✘	–	–	–	–	–	–	–	–
Haz	✓	✓	✓	✘✓	✘✘✓	✘✘✘	–	–	–
Ido	✘✓	✓	✘✓	✘✓	✘✘✘	–	–	–	–
Josh	✓	✘✘✓	✘✘✘	–	–	–	–	–	–

(c) List, in order, the athletes who came 1st, 2nd, 3rd, 4th and 5th. [2]

(d) **(i)** State the greatest distance that Ken could have jumped successfully, and, for this case, state all his possible total numbers of fails. [2]

(ii) State the least distance that Ken could have jumped successfully, and, for this case, state all his possible total numbers of fails. [2]

These 11 athletes took part in a second event in the same competition. A summary of the greatest distance that each jumped successfully and their total numbers of fails is shown below.

Athlete	Adi	Ben	Cal	Den	Eric	Fran	Greg	Haz	Ido	Josh	Ken
Distance	5.6	5.8	5.4	5.2	5.4	5.2	5.8	6.0	6.0	5.2	6.2
Total fails	7	8	6	9	5	5	7	5	6	8	3

(e) What can you deduce about the greatest distance that could have been set by the organisers for the first round of this event? Explain your answer. [2]

In order to find the overall winner of the two-event competition, points are awarded in each of the two events: 11 points for first place, 10 points for 2nd place, 9 points for 3rd place and so on to 1 point for 11th place. The points for each of the two events are added to give the athlete's total number of points. The athlete with the highest total number of points is the winner. If there is a tie between athletes, the one with the fewest total number of fails in the two events is the winner.

(f) Which athletes finished 1st, 2nd and 3rd overall in this two-event competition? Justify your answer. [3]

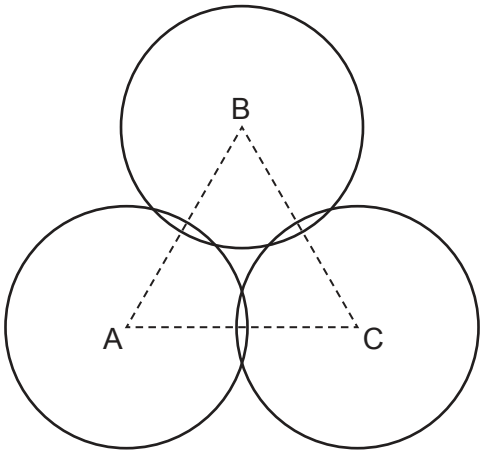
3 Every year, on the island of Apodidia, the Birds In Real Danger Service (BIRDS) attempts to estimate the number of swifts on the island. In order to do this they ask residents to make records of the number of birds of that species they see. They are asked to synchronise their clocks and then survey the sky at the beginning of each of six successive minutes: they record the time and the number of birds they can see at that moment.

BIRDS asks those surveying to report only birds that they are certain are swifts. The maximum distance at which an observer can identify a swift with certainty is 1 km.

The following data was reported by three observers, Alana, Barbara and Carla. (A blank cell means that that person did not take a reading at that time.)

Time	*Alana*	*Barbara*	*Carla*
11:00	8	0	
11:01		6	3
11:02	10	10	10
11:03	2	12	12
11:04	13	1	
11:05	9	4	9

The observers also sent in their positions, which formed the corners of an equilateral triangle, just under 2 km apart, as shown on the diagram. The circles show the 1 km range within which swifts can be identified with certainty.



(a) **(i)** What is the largest number of birds that could have caused the reports given by Alana and Barbara? [1]

(ii) What is the smallest number of birds that could have caused the reports given by Alana and Barbara? [1]

(b) What is the smallest number of birds that could have caused the reports given by all three observers? Explain your answer. [3]

A BIRDS official discovers that one of these three observers cannot distinguish a swift from a different bird, a swallow.

(c) What is the smallest number of swifts that could have caused the reports, given this new information? [1]

The same official also discovers that one of the other observers was using binoculars; as a result, they could identify with certainty all swifts up to 3 km away. This observer was not the one who cannot distinguish a swift from a swallow.

(d) Explain how it can be deduced which of the observers must have had binoculars, and which one could not distinguish a swift from a swallow, based on the data reported. [4]

[Turn over for Question 4]

4 Jessica runs a small business which makes wooden puzzles. Jessica employs Graham to make the puzzles.

The materials to make one puzzle cost $44 and it takes Graham 30 minutes to make each puzzle. Jessica pays Graham at a rate of $12.00 per hour (e.g. $97 for 8 hours 5 minutes worked).

Jessica works out the total cost to produce one puzzle as the cost for the materials plus the amount paid to Graham to make the puzzle. She does not include any other costs that are incurred. Jessica decides that the selling price for a puzzle will be 20% more than the total cost to produce it.

(a) What will be the selling price of one puzzle? [1]

The puzzles are very popular, so Jessica also employs Tom to make puzzles. She pays Tom at a rate of $12.00 per hour.

Tom takes 25 minutes to make a puzzle, but he sometimes makes a mistake. If he makes a mistake then the puzzle is given to Graham to correct. This requires additional materials costing $10. It takes Graham 15 minutes to correct each puzzle.

Jessica now decides that she will set the selling price of the puzzles at $65 each.

(b) (i) What is the greatest profit that Jessica could make from selling one puzzle? [1]

(ii) What is the least profit that Jessica could make from selling one puzzle? [1]

Graham and Tom both work five days each week, from Monday to Friday. Tom starts work at 08:00 each day and Graham starts work at 09:00. Throughout the day, Graham always corrects any mistakes that Tom has made before beginning to make a puzzle himself.

Graham and Tom both work for at least 8 hours in a day. They each take an unpaid one-hour break as soon as they complete the puzzle that they are working on 4 hours into their day. Once they have worked for 8 hours they do not begin work on a new puzzle, but they do complete any puzzles that have been started before they leave. Graham also makes sure that any of Tom's mistakes have been corrected before he leaves.

(c) What is the latest time that Graham might start work on his first new puzzle on any day? [2]

(d) At what time does Tom finish work each day? [2]

(e) (i) What is the most profit that could be made for a day's production? [2]

(ii) What is the least profit that could be made for a day's production? [2]

Graham complains to Jessica that it is not fair that he is paid at the same rate as Tom. Jessica decides to regard the profits made from puzzles as having been generated by either Graham or Tom: if Tom makes a puzzle without mistakes then he is regarded as having generated the profit for that puzzle, while Graham will be regarded as having generated the profits for all of the other puzzles.

Jessica will set the new rate of Graham's pay so that, on a day in which exactly half of the puzzles made by Tom needed to be corrected, she would regard Tom and Graham as having generated the same amount of profit.

(f) What is the hourly rate of pay that Graham will now receive? [4]